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THE PHYSICAL EPHEMERIS OF MARS

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PREFACE

Gerard de Vaucouleurs, Professor of Astronomy at the University of Texas and a consultant to The RAND Corporation, has undertaken a long-term project to produce a map of the surface features of Mars. The goal is a map that is based on sound data, derived by the examination and statistical correlation of all available areographic records published since 1877.

To correlate and combine recorded observations of Martian features, the locations on the planetary disk must be reduced to areocentric coordinates. This reduction requires a Martian physical ephemeris, the accuracy of which directly affects the accuracy of the reduction. Observers since 1877 have used various ephemerides of varying degrees of accuracy. In this Memorandum, Dr. de Vaucouleurs details the method for arriving at his adopted values for the constants on which the new physical ephemeris will be based. With the new ephemeris, he proposes to reduce all available observational data to a uniform homogeneous system. He also discusses the other values for the basic constants that have been used in the past.

ABSTRACT

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A revision of the physical ephemeris of Mars is one of the first

steps toward a reasonably accurate Martian map. The complexities

in determining true values are pointed out, and a set of values is

found that serve as the basis for computing the new ephemeris.

A U T H O R

CONTENTS

THE PHYSICAL EPHEMERIS OF MARS

A physical ephemeris of Mars provides the elements needed for the reduction of measurements of observed (projected) coordinates of surface markings to a system of planetocentric (areocentric) coordinates. These elements are:

(1) the position angle, p, of the central meridian at the time of the observation, i.e., of the projection of the rotation axis on to the plane tangent to the celestial sphere;

(2) the areocentric latitude, D_E, of the center of the planet's disk, i.e., the declination of Earth referred to the equator of Mars (Earth transits at the zenith of points of latitude D_E on the planet);

(3) The areographic longitude of the central meridian, ω, reckoned from some arbitrary meridian.

The first two elements, p and D_E, depend on the orbital elements of Earth and Mars, and on the relative positions of both planets on these orbits, i.e., on Ephemeris Time; they depend also on the direction of the axis of rotation of Mars, i.e., on the celestial coordinates $M_0(\alpha_0, \delta_0)$ of the North Pole of Mars. The third element depends, further, on the adopted value ω_0 of ω at some arbitrary time $t = t_0$, and on the sidereal rotation period of Mars, P (or its reciprocal $R = 360^\circ/P$, the angular velocity of rotation).

For a detailed discussion of the relations between these elements reference is made to a paper by K. Graff[1] of which an English translation was recently given in a RAND Memorandum.[2] Only the formulae needed in the following discussion will be repeated here in a simpler notation and with additional details on some important points not covered by Graff.
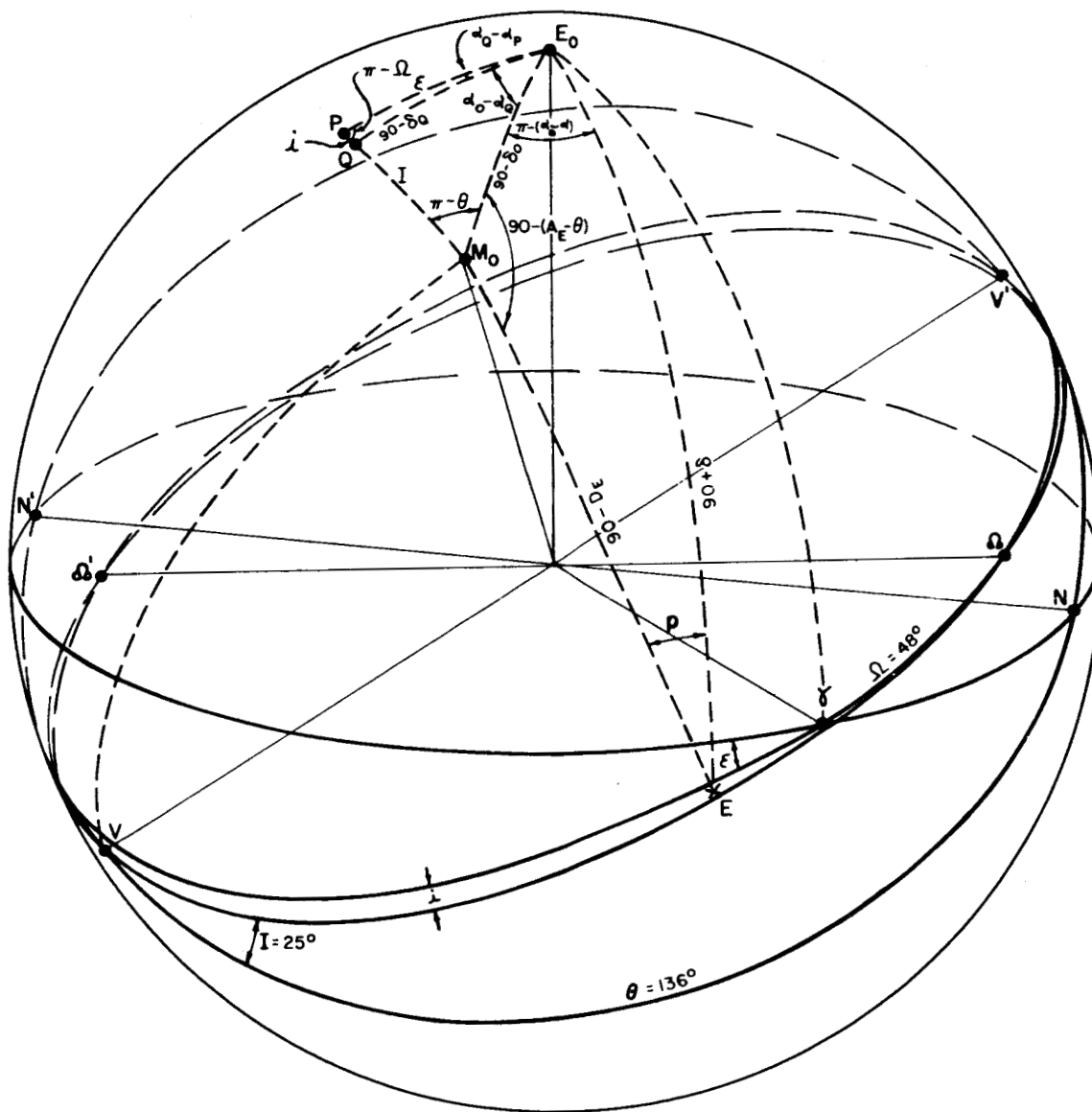
Fig. 1 Celestial sphere showing orbital planes,
equators and poles of Earth and Mars

The angle ϱ is found by solving the spherical triangle $(E_0 M_0 Q)$ in which one angle and two sides, namely

$$\hat{E}_0 = \alpha_0 - \alpha_Q, \quad (E_0 Q) = 90^\circ - \delta_Q, \quad (E_0 M_0) = 90^\circ - \delta_0,$$

are known, once the equatorial coordinates of Q are known. Then $\hat{M}_0 = 180^\circ - \varrho$ and the inclination of the equator of Mars to the plane of the orbit $(M_0 Q) = I$ are given by

$$\begin{cases} \sin I \sin (\pi - \varrho) = \cos \delta_Q \sin (\alpha_0 - \alpha_Q) \\ \sin I \cos (\pi - \varrho) = -\sin \delta_Q \cos \delta_0 + \cos \delta_Q \sin \delta_0 \cos (\alpha_0 - \alpha_Q) \\ \cos I = \sin \delta_Q \sin \delta_0 + \cos \delta_Q \cos \delta_0 \cos (\alpha_0 - \alpha_Q). \end{cases} \quad (2)$$

The equatorial coordinates of Q, in turn, are found by solving the spherical triangle $(E_0 P Q)$, in which one angle and two sides, namely

$$\hat{P} = 180^\circ - \Omega, \qquad (PQ) = i, \qquad (E_0 P) = \epsilon,$$

are known with the orbital elements Ω, i, and the obliquity of the ecliptic, ϵ. Then the angles

$$\hat{E}_0 = \alpha_Q - \alpha_P = 90^\circ + \alpha_Q \text{ and } (E_0 Q) = 90^\circ - \delta_Q$$

are given by

$$\begin{cases} \cos \delta_Q \cos \alpha_Q = \sin i \sin \Omega \\ \cos \delta_Q \sin \alpha_Q = \cos i \sin \epsilon + \sin i \cos \epsilon \cos \Omega \\ \sin \delta_Q = \cos i \cos \epsilon - \sin i \sin \epsilon \cos \Omega . \end{cases} \quad (3)$$

Next consider ω. We will need to discuss several related quantities, which should be carefully distinguished.

The symbol ω is the areographic longitude of the central meridian at a particular time t. At some origin of time, $t = t_0$, the areographic longitude of the central meridian is ω_0 measured from some conventional

However, the pole of Mars, M_0, itself is not fixed in the celestial sphere because of a small (solar) precession of the Martian equinoxes, estimated by H. Struve at μ = 7".07 per year.[4] The precession causes M_0 to move in a retrograde direction about Q in a circle of radius I (Fig. 3); the rate of precessional motion is μ sin I (seconds per year), of which the component μ_δ = $-\mu$sin I sin $(\pi - \theta)$ is in declination, and the component μ_α cos δ_0 = $-\mu$sin I cos $(\pi - \theta)$ is in right ascension. Hence the effect of the Martian precession is to diminish the (terrestrial) precession corrections to the celestial coordinates of M_0; the total corrections then are

$$d\alpha_0/dt = [m + n \sin \alpha_0 \tan \delta_0] - [\mu\sin I \cos (\pi - \theta) \sec \delta_0] = A - B$$
$$d\delta_0/dt = [n \cos \alpha_0] \qquad\qquad - [\mu\sin I \sin (\pi - \theta)] \qquad = C - D.$$

$$(6a)$$

For the adopted values of α_0, δ_0, and μ, the values of A,B,C,D are

$$A = 0°.00775, \quad B = 0°.00100, \quad C = 0°.00403, \quad D = 0°.00057,$$

and the total corrections are given by

$$\alpha_0(t) = \alpha_0(t_0) + 0°.00675 (t - t_0)$$
$$\delta_0(t) = \delta_0(t_0) + 0°.00346 (t - t_0),$$

$$(6b)$$

with sufficient accuracy when $|t - t_0|$ < 100 years. The angle θ is also affected by the differential precession of the equinoxes of Earth and Mars; for the adopted values of α_0, δ_0, and μ, and again for moderate time intervals, the annual motion is given with sufficient accuracy by

$$d\theta/dt = - 0°.00538 (t - t_0).$$

$$(6c)$$

The values of α_0, δ_0 (Equinox 1905.0) adopted in various ephemerides are listed in Table 1. The values used by Marth for the years 1869 to 1877 were after Bessel and Oudemans, for 1879 to 1883 and 1886 to 1894 after Schiaparelli, and those used by Marth for 1896 and by Crommelin for 1898 to 1905 after H. Struve; since 1909 the American Ephemeris and the British Nautical Almanac have used values derived by Lowell and Crommelin in 1905 from several sources.[5]

The values of α_0, δ_0 included in Lowell's discussion are listed in Table 2 together with more recent determinations by various methods. These methods are, briefly,

(a) observations of the polar caps, with allowance for the slight eccentricity of the caps with respect to the areographic poles (Lowell 1905; Wirtz 1912; Widorn 1939).

(b) observations of the apparent elliptical paths of surface details in the diurnal rotation (Trumpler 1927; Camichel 1954),

(c) determinations of the pole of the mean orbital planes of the satellites (Struve 1911; Burton 1929).

The data are in good agreement in right ascension, but there is an unexplained discrepancy of $1°.4$ in declination between the means of methods (1--4) and (5,6). Because of the good agreement of the latter, which are completely independent, and because a *priori* they would seem to be the least subject to systematic error, their unweighted means, rounded off to the nearest $0°.05$ (Table 2b), namely,

$$\alpha_0 = 316°.55 \qquad \delta_0 = +52°.85 \qquad (1905.0),$$

(d) In M.N., 46, 29, 1886, Marth introduces a correction to the pole after Schiaparelli's observations of 1877--79, taking for 1880.0, i = 36°380; n = 48°130; i.e., α_0 = 318°130; δ_0 = +53°62. Further, ω_0 was corrected by +1°32 to make the longitude of Fastigium Aryn = 0° in Schiaparelli's observations.

(e) In M.N., 56, 394, 1896, Marth mentions but does not specify a change in ω_0 to fit Lowell's observations; it is not clear whether a new rotation rate was adopted. Marth states "an alteration of the rotation rate ... could not be reconciled with the older observations," but see note (f).

(f) In M.N., 58, 468, 528, 1898, Crommelin states that his ephemeris is based on Marth's new constants for 1896, which seems to contradict Marth's comments in note (e). An a posteriori comparison of Marth's 1896 ephemeris with the revised ephemeris confirms that a set of new constants had been used which are consistent with Crommelin's ephemeris.

(g) In the Appendix, p. 48, to the Nautical Almanac for 1909 the adoption of Lowell's pole is noted with the comment "As the adoption of a new position for Mars' North Pole involved a discontinuity in that ephemeris, the longitude at the beginning of the ephemeris has been made the same as if the former value had been continued."

(h) Ephemeris Time.

(i) The adoption of Ashbrook's revised rotation period without corresponding change in ω_0 introduces a discontinuity of about 1° in ω at the beginning of 1960.

Sources

1. M.N., 29, 53, 1869.
2. M.N., 37, 301.
3. M.N., 39, 468.
4. M.N., 43, 490.
5. M.N., 46, 29.
6. M.N., 48, 78.
7. M.N., 50, 127.
8. M.N., 52, 398.
9. M.N., 54, 394.
10. M.N., 56, 394.
11. M.N., 58, 468, 528.
12. M.N., 60, 323.
13. M.N., 62, 604.
14. M.N., 64, 506.
15. Nautical Almanac or American Ephemeris, 1909 to 1959.
16. Nautical Almanac or American Ephemeris since 1960.
17. Revised ephemeris computed by American Ephemeris Office for Mars Map Project (see text).

Table 2b

MEAN VALUES FOR THE CELESTIAL COORDINATES OF

THE NORTH POLE OF MARS GROUPED ACCORDING TO METHOD

Data	α_0	δ_0
Adopted by Nautical Almanac since 1909 from method 1 data for 1877--1905 (lines 1 to 5, Table 2a).	317°.50	54°.50
Mean of additional method 1 data for 1901--1926(lines 6 to 12, Table 2a).	315.52	54.01
Mean of all method 1 data.	316.51	54.25
Mean of method 2 and 3.	317.12	53.78
Method 4	315.77	54.63
Mean of methods 2, 3, and 4.	316.45	54.20
Method 5	316.48	52.78
Method 6	316.60	52.94
Adopted average of methods 5 and 6.	316°.55	52°.85

Table 3

ROTATION PERIOD OF MARS ACCORDING TO FOUR AUTHORS

Year of Publication	Author	Method	$P = 24^h 37^m +$	m.e.	Source	Notes
1886	Wislicenus	Drawings 1659--1881	22.655	.013	(1)	(a, b)
1897	Bakhuyzen	Drawings 1659--1881	22.66	.0132	(2)	(b)
1953	Ashbrook	Transits 1879--1952	22.6689	.0026	(3)	(c, d)
1953	Ashbrook	Drawings, Syrtis Major 1659--1881	22.672	.018	(3)	(c, e)

Notes

(a) m.e. recomputed by Ashbrook (source 3 below).

(b) True rotation period, mean solar time. Mean solar time unit is essentially identical with Ephemeris Time over period considered.

(c) Sidereal rotation period, Ephemeris Time.

(d) True rotation period is $0^s.0012$ longer. Rotation period in mean solar time is $0^s.0010$ shorter over period considered.

(e) Given as consistency check only to verify agreement of historical drawings with modern transits.

Sources

(1) Wislicenus, W., "Beitrag zur Bestimmung der Rotationszeit des Planeten Mars," Strassburg Inaugural Dissertation, Karlsruhe, 1886.

(2) Bakhuyzen, H. G. van de Sande, "Untersuchungen über die Rotationszeit des Planeten Mars," Ann. Sternw. Leiden, 7, 1, 1897; abstract in Vierteljahrschrift Astron. Gesellsch., 20, 237, 1885.

(3) Ashbrook, Joseph, "A New Determination of the Rotation Period of the Planet Mars," A.J., 58, 145, 1953.

Through the courtesy of the successive directors of the American Ephemeris Office, Drs. G. C. Clemence and R. L. Duncombe, a new physical ephemeris of Mars was recently computed at the U.S. Naval Observatory for all oppositions from 1877 to 1965. The following elements were adopted:

Zero point of longitudes: $\omega_0 = 344°.41$ on JD 2418322.0, being the value adopted by the ephemerides since 1909 (and conforming to the earlier ephemeris back to 1896) (see Table 1);

North Pole of Mars: $\alpha_0 = 316°.55$, $\delta_0 = +52°.85$ (Equinox 1905.0) being the mean of determinations by Burton (1929) and Camichel (1956) (see Table 2);

Sidereal rotation rate: $R = 350°.891962$ per day (E.T.), corresponding to the rotation period derived by Ashbrook (1953) and adopted by the Ephemeris since 1960 (see Table 3).

A number of fundamental and derived constants in the N.A.1 system and in the revised system (neglecting nutation and secular terms $<0°.01$ per century) are listed in Table 4. These data are now being used in the Mars Map Project[7] to reduce all available areographic-coordinate data since 1877[8,9] to a uniform and homogeneous system.

REFERENCES

1. Graff, K., "Beiträge zur physischen Untersuchung der grossen Planeten. 1. Beobachtungen und Zeichnungen des Planeten Mars während der Oppositionen 1901 und 1909," Astronomische Abhandlungen der Hamburger Sternwarte, Bergedorf, Band 2, Nr. 1, pp. 1-23, 1910.

2. Graff, K., Derivation of the Formulae for the Reduction of Planetographic Observations with Special Reference to the Planet Mars (C. H. Michelis, trans.), RM-3138-NASA, The RAND Corporation, June 1962.

3. de Vaucouleurs, G., The Astronomical Unit of Distance. Solar Parallax and Related Constants, RM-2944-NASA, The RAND Corporation, December 1961.

4. Struve, H., Astron. Nach., 138, Nr. 3302, 217, 1895.

5. Lowell, P., M.N.R.A.S., 66, 54, 1905.

6. Ashbrook, Joseph, "A New Determination of the Rotation Period of the Planet Mars," A.J., 58, 145, 1953.

7. de Vaucouleurs, G., "Precision Mapping of Mars," in La Physique des Planètes, Bull. Soc. Roy. Sci., Liège, 24, 369, 1962.

8. de Vaucouleurs, G., and R. Wright, Sources of Areographic Coordinates, 1909-1954, Harvard Observatory Report 2, AFCRL Report 257, February 1961.

9. de Vaucouleurs, G., and R. Wright, Sources of Areographic Coordinates, 1877-1907, RM-3991-NASA, The RAND Corporation, in press.